

2022 International Summer Courses on Analysis Algebra and Computation



Summary Report

China Nanjing

Aug. 8 Sep.2, 2022

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COURSE 1: SELECTED TOPICS IN MODERN MATHEMATICS

Hours/Credits	24 hours (August 8 - September 2, 2022) / 1 credit	
	Tues. 18:30-20:55, Thu. 18:30-20:55	
	Onsite + Online Platform: Zoom + QQ	
Description	After this course, students should learn how to formul variational problems and be able to apply the Calculus of Variations to a range of minimization problems in physics and mechanics	
	The calculus of variations concerns problems in which one wishes to find the extrema (usually the minima) of some quantity over a system that has functional degrees of freedom. Many important problems arise in this way across pure and applied mathematics and physics. In this course it is shown that such variational problems give rise to a system of differential equations, the Euler-Lagrange equations. These equations, which have far reaching applications, and the techniques for their solution, will be studied in detail.	
Instructor	Professor Alastair Rucklidge Leeds University Homepage: http://www1.maths.leeds.ac.uk/~alastair/	
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PREREQUISITES		

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CLASS SCHEDULE	
Day	Contents
Aug. 9	Introduction; The Euler Lagrange equation;
Aug. 11	The Brachistochrone;
Aug. 16	The Propagation of Light Rays ;
Aug. 18	Extensions of the Basic Theory;
Aug. 23	Calculating the second variation; C e i j Zfe kfe;
Aug. 25	Constrained Problems; Isoperimetric problems
Aug. 30	The hanging chain; Local constraints;
Sep. 1	Final exam.
FEEDBACK FROM STUDENTS	

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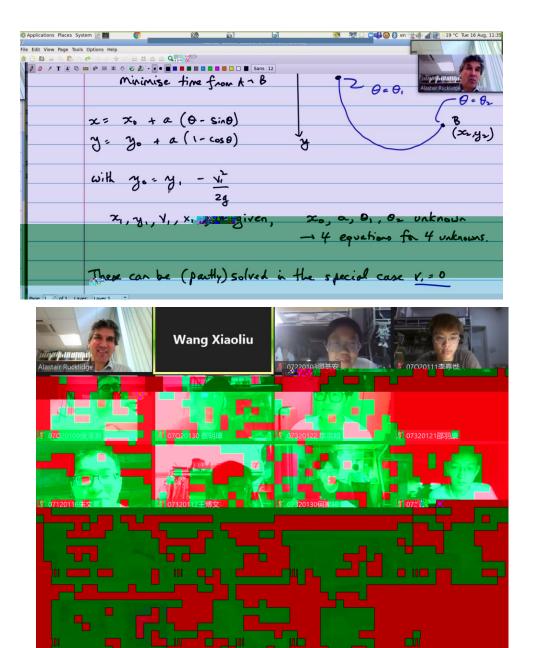
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FEEDBACK FROM TEACHERS

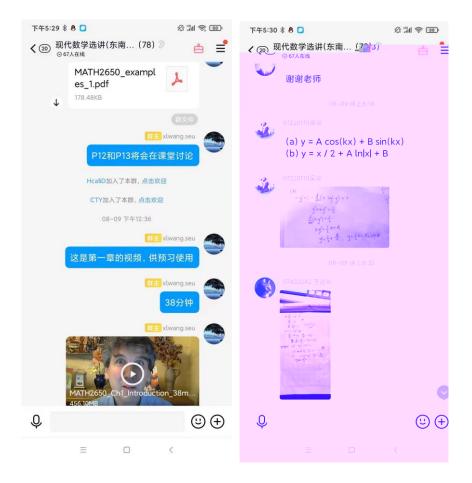
Alastair Rucklidge

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COURSE 2: SELECTED TOPICS IN ADVANCED ALGEBRA

Hours/Credits	32 hours (August 8 - September 2, 2022)/ 1.5 credit
	Mon. 14:00-16:35, Wed. 18:30-20:55, Fri.14:00-16:35 (online &
	Mon. 14:00-16:35, Wed. 18:30-20:05 for last week)
	Platform: Tencent Meeting + QQ
Description	In this course, the students will be introduced to the basics of the theory of generalized inverses, the field that has grown much in the last years and is still growing. A lot of illustrations of the theory will be presented with applications in many areas. We will provide an overview of different classes of generalized inverses, their characterizations, different presentations and properties as well as their applicability to different problems inside and outside of mathematics. After the course, students will be familiar with the basic knowledge of the theory of generalized inverses and its possible applications,
	and be able to follow certain more advanced topics from this field.
Instructor	; i e J : mkbfm - c Le mijkpf E [·] Homepage: https://www.pmf.ni.ac.rs/Dragana/index.html
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	Proc. Amer.
	Math. Soc. Act a Math. Sci. Linear Algebra Appl.
	App. Math. Comp. Linear & Multilinear Algebra
Prof. Dragana S	. 80
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PREREQUISITES

Only elementary knowledge of linear algebra is assumed.

COURSE OBJECTIVES

After this course, students should

- be familiar with the basic knowledge of the theory of generalized inverses and its possible applications,
- be able to follow certain more advanced topics from this field.

CLASS SCHEDULE

Day	Contents
August 8	Historical note. Preliminaries: vector spaces, linear transformations, matrix representations, full-rank factorization, idempotents and projectors, adjoints
August 10	Different classes of generalized inverses. Construction of {1}-inverses and their properties. Existence and construction of {1, 2} inverses. Existence and construction of {1, 2, 3}, {1, 2, 4} and {1, 2, 3, 4} inverses
August 12	Explicit formula for Moore-Penrose inverse. Construction of {2} inverses of prescribed rank. Diversity of generalized inverses and their different characterizations
August 15	Applications: Solvability of different linear systems and representations of their solutions. An application of {2} inverses in iterative methods for solving nonlinear equations. A {1, 2} inverse for the integral solution of linear equations
August 17	The Bott Duffin inverse. An applicatio

FEEDBACK FROM STUDENTS

COMMENT 1

After the study in summer course, as well as reading the relevant literature and books review I learned about the concept of generalized inverse, and had a certain understanding of generalized inverse theory and related properties, which put me into the wide range of knowledge involved in the field of higher algebra, and also arouse my more interest in the field of algebra. At the same time, my logical reasoning ability, which is embodied in the proof of the theorem and lemma, has also been improved to some extent.

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In this course, we learned several definitions of generalized inverses

described the properties that the inverse of an element should have by several equations, just like what we have learnt in physics. By adding equations he found the only one point in the space of linear transformations that satisfy the properties he mentioned.

COMMENT 4

Through I earning this course, I began to think about how new theorems were conceived. In addition to some that can be deduced from the original theorems, we can remove a condition of the original theorem to observe whether the proposition is tenable, and consider whether new conditions can be added to get a better theorem. In this way, new discoveries may be made by disassembling or adding conditions to the original theorem My way of thinking has been improved through this course.

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In fact, the study of this course is not very easy for me because of the English teaching. Fortunately, Mr. Zhang explained in Chinese every time after Ms. Dragana finished lecture, which greatly helped me to understand what I had just learned. In addition, after each class, I can get the course screen recording and the teacher's manuscript so that I could review the knowledge again and again after the class to deepen my grasp of what I learned.

The study of this course made me appreciate more of the charmof Algebra. There are many knowledge about Algebra, and its application is very wide.

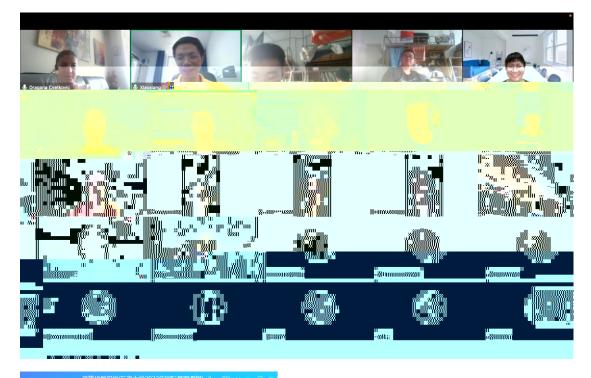
In the future study and life, I would also like to learn and understand more and more relevant knowledge to enrich myself.

COMMENT 8

This course, due to its short duration and high difficulty, requires not only to follow the course to study, but also to consult some materials by myself. In the process, I greatly exercised my self-learning ability. For example, some commonly used book resource websites and some techniques for searching keywords are gradually mastered in such a little exploration. In the future study, we may face more difficult problems, and face more open problems, which cannot be solved according to the script. I think this is a good start and will be of great help to future research work.

This course is a small part of the follow-up to advanced algebra, and it also triggered my thinking about the direction of personal follow-up study. Recently, I have also begun to consider seriously the direction of study. Combined with some of my extracurricular learning, I feel more interested in the direction of reinforcement learning.

FEEDBACK FROM TEACHERS



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COURSE 3: SELECTED TOPICS IN FRONTIER OF SCIENTIFIC COMPUTATION

Hours/Credits	24 hours (August 8 - September 2, 2022) / 1 credit
	Mon. 9:50 12:15, Wed. 9:50 12:15
	Online Platform: Zoom + QQ
Description	The main purpose and characteristic feature of this program is the accessibility of presentation and an attempt to cover the rapidly developing areas of the theory,

First publications on inverse and ill-posed problems date back to the first half of the 20th century. Their subjects were related to physics (inverse problems of quantum scattering theory), geophysics (inverse problems of electrical prospecting, seismology, and potential theory), astronomy, and other areas of natural sciences. Since the advent of powerful computers, the area of application for the theory of inverse and ill-posed problems has extended to almost all fields of science that use mathematical methods.

numerical methods and applications of inverse and

ill-posed problems as completely as possible.

direct problems of mathematical physics, In researchers try to find exact or approximate functions that describe various physical phenomena such as the propagation of sound, heat, seismic waves, electromagnetic waves, etc. In these problems, the media properties (expressed by the equation coefficients) and the initial state of the process under study (in the nonstationary case) or its properties on the boundary (in the case of a bounded domain and/or in the stationary case) are assumed to be known. However, it is precisely the media properties that are often unknown. This leads to inverse problems, in which it is required to determine the equation coefficients from the information about the solution of the direct problem. Most of these problems are ill-posed (unstable with respect to measurement errors). At the same time, the unknown equation coefficients usually represent important media properties such as density, electrical conductivity, heat conductivity, etc. Given such a wide variety of applications, it is no surprise that the theory and numerical methods of inverse and ill-posed problems has become one of the most rapidly developing areas of modern science. Today it is almost impossible to estimate the total number of scientific publications that directly or indirectly deal with inverse and ill-posed problems. However, since the theory, numerical methods are relatively young, there are many terms are still not well-established, many important results are still being discussed and attempts are being made to improve them. New approaches, concepts, theorems, methods, algorithms and practical problems are constantly emerging.

The calculus of variations concerns problems in which one wishes to find the extrema (usually the minima) of some quantity over a system that has functional degrees of freedom. Many important problems arise in this way across pure and applied mathematics and physics. In this course it is shown that such variational problems give rise to a system of differential equations, the Euler-Lagrange equations. These equations, which have far reaching applications, and the techniques for their solution, will be studied in detail.

Instructor	Prof. Maxim A. Shishlenin (Institute of Computational Mathematics and Mathematical Geophysics, RAS) Homepage: https://icmmg.nsc.ru/ru/content/employees/ shishlenin-maksim-aleksandrovich
	Maxim A. Shishlenin 2003

Journal of Inverse and III-posed Problems

Numerical Analysis and Applications, Eurasian Journal

of Mathematical and Computer Applications, Siberian Electronic Mathematical Reports

PREREQUISITES

Shishlenin

Calculus, Linear Algebra, Differential Equations, Numerical Analysis. Students are strongly encouraged to use MATLAB for programming.

COURSE OBJECTIVES

Prof. Maxim A.

After this course, students should be able to

- understand the concept of ill-posed and inverse problems;
- master the regularization methods for inverse problems;
- understand the applications of ill-posed and inverse problems.

CLASS SCHEDULE

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